The Lanchester Square Law: Its Implications for Force Structure and Force Preparation of Singapore's Operationally-Ready Soldiers

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Abstract:

In this essay, the author explains the Lanchester Square Law Theory and links it to the concept of Relative Combat Power (RCP). RCP is defined as the effectiveness of a force in killing an enemy. The author shows the implications of the Lanchester Square Law for tactics in combat, by exploring test-cases of battles between a seven-man and ten-man section. Next, he explains how Admiral Lord Nelson's tactics at the Battle of Trafalgar were in fact an application of the Lanchester Square Law. He feels that the importance of numbers has implications for the force structure of our army and the advantage of an opponent's numbers needs to be compensated by a significant increase in the quality of our own forces. In his opinion, the commanders should maintain a high baseline of quality in peacetime, as Singapore depends heavily on both the quantity and quality of our Operationally-Ready servicemen. Only then can the SAF harness the advantages of quantity and quality, enhanced by detailed and coordinated planning, to secure a swift and decisive victory over any aggressor.

Keywords: Quantity and Quality; Outcome of Battle; Survival; Tactics; Force Preparation

INTRODUCTION

> *"He will win who knows when to fight and when not to fight;*
>
> *he will win who knows how to handle both superior and inferior forces..."*
>
> *- Sun Zi*[1]

In September 2015, an eight-man Special Air Service (SAS) team was ambushed in Syria by at least 30 militants, while smuggling a secret agent into Syria. The SAS team was "out-gunned and out-numbered" but regained the initiative "by using courage, aggression and firepower."[2] In fact, one SAS soldier outgunned six militants. Eventually, the SAS team eliminated the ambush and killed eight militants.[3]

Does the quality of soldiers matter more than their quantity? In this essay, the author explains the Lanchester Square Law and links it to the concept of Relative Combat Power (RCP). RCP is defined as the effectiveness of a force in killing an enemy. The author shows the implications of the Lanchester Square Law for tactics in combat, by exploring test-cases of battles between a seven-man and ten-man section. Next, he explains how Nelson's tactics at the Battle of Trafalgar were in fact an application of the Lanchester Square Law. Finally, the author concludes by highlighting: (1) the importance of numbers and implications for our Army's force structure; and (2) the importance of quality and implications for force preparation of our operationally-ready forces.

THE LANCHESTER SQUARE LAW AND RCP

According to the author, the Lanchester Square Law allows us to compare the RCP of two fighting forces and anticipate the outcome of battle. In this section, the author highlights three points: (1) RCP of a force is not the number of units; but (2) the RCP is proportional to the square of the number of units and proportional to the quality of units; and (3) if we know the qualities and numbers of two forces at the start of a battle, we can tell the outcome.[4]

Consider two forces, Red and Blue, engaged in combat. Each soldier knows the locations of their targets and shifts fire to a new target when the previous target is destroyed. Let functions R(t) and B(t) represent the numbers of Red and Blue forces fighting at any time t, where t>0. The original numbers of Red and Blue forces are R(0) and B(0). Let the quality of each unit of forces be defined by the number of enemy units it destroys per unit time, where each unit of Red forces destroy r blue units per unit time, and each unit of Blue forces destroy b red units per unit time.

Then, the rate of Red forces being destroyed is:

$$dR/dt=-bB \quad [1]$$

Similarly, the rate of Blue forces being destroyed is:

$$dB/dt=-rR \quad [2]$$

Dividing the two equations to eliminate t , we obtain:

$$dB/dR=rR/bB \quad [3]$$

Cross-multiplying both sides gives:

$$bBdB=rRdR \quad [4]$$

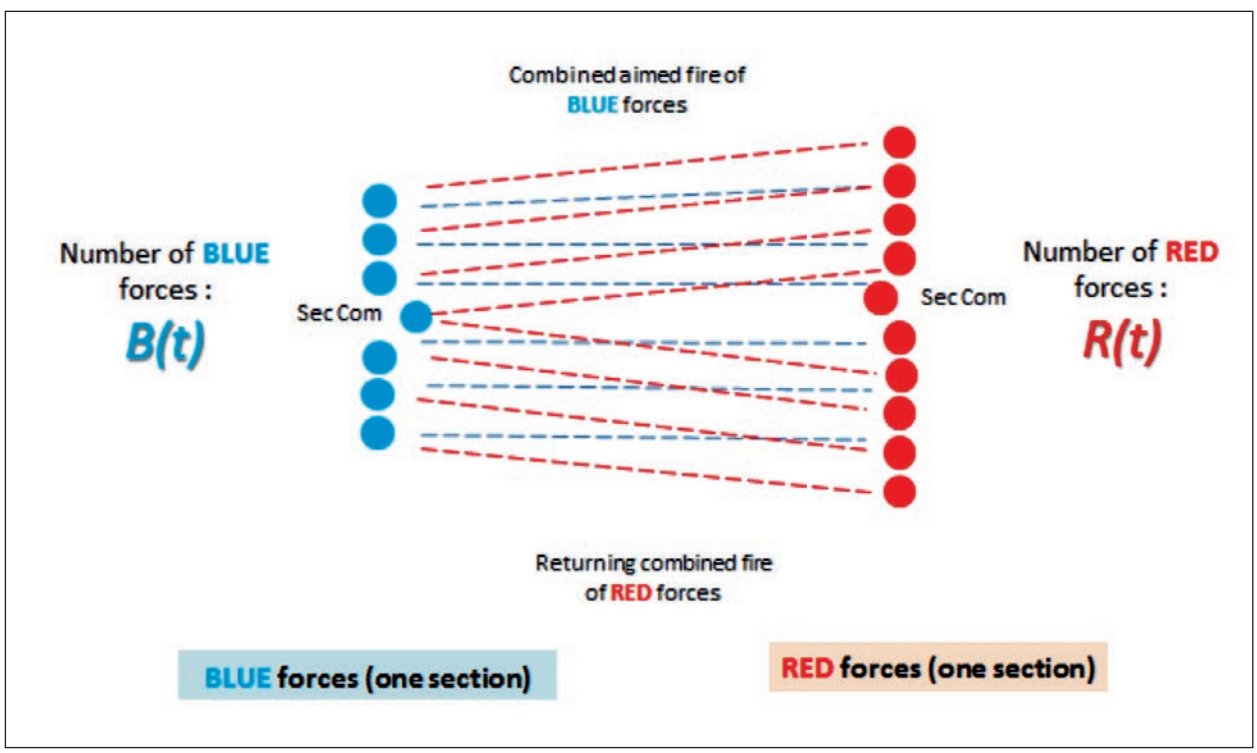

Figure 1: Illustration of Battle between R(t) Numbers of Red Forces and B(t) Numbers of Blue Forces.

Integrating both sides over all time t, from t=0 to arbitrary time t:

$$b\int BdB=r\int RdR \quad [5]$$

$$bB^2(t)-rR^2(t)=bB^2(0)-rR^2(0) \quad [6]$$

Note that the difference between $bB^2(t)$ and $rR^2(t)$ is the same for all time, because t can represent any time as long as $t\geq 0$. So, we can claim that the difference $bB^2(t)-rR^2(t)$ equals constant K:

$$bB^2(t)-rR^2(t)=bB^2(0)-rR^2(0)=K \quad [7]$$

Equation (Eq.) [7] is the Lanchester Square Law: the difference $bB^2(t)-rR^2(t)$ is a constant all the time. K remains the same throughout the battle, whether at the start (at t=0), or at any later time t at the end. This means that if Blue forces begin with $bB^2(0)>rR^2(0)$, they will definitely win at the end because $bB^2(t)$ will always be greater than $rR^2(t)$.[5]

The K value determines which force wins. If K=0, then:

$$bB^2(t)=rR^2(t) \quad [8]$$

This is the case that both forces are equally-matched, and fight till both are eliminated.

If K>0, then:

$$bB^2(t)>rR^2(t) \quad [9]$$

This is the case that Blue forces win, when $bB^2(0)>rR^2(0)$ at the start of the battle. If $K<0$, then:

$$rR^2(t) > bB^2(t) \quad [10]$$

This is the case that Red forces win, when $rR^2(0) > bB^2(0)$ at the start of the battle.

The Lanchester Square Law thus gives the following results: $rR^2(t)$ and $bB^2(t)$ represent the RCP of each force that measures how effectively a force attrites another. The RCP of a force is not the number of units. Instead, the RCP is proportional to the *square* of the number of the units, and proportional to the quality of the units. Moreover, from Eq. [7] we know that the difference between the RCP of two forces is equal to a constant K. Therefore, if we know the numbers and qualities of two forces at the start of a battle, we can anticipate which side will win.

APPLICATION TO SIMPLE CASES OF COMBAT

In this section, the author applies the results of the discussion above to model two simple cases of combat. Do note that the numbers used are hypothetical and are used for illustration only.

Test Case 1

Consider a test-case where both Blue and Red soldiers are of the same quality, with a seven-man Blue infantry section against a ten-man Red section. Let both forces have quality q. We want to determine which side wins and with how many surviving troops.

At the start, the RCP bB^2 of the seven-man Blue infantry section is:

$$bB(0)^2=q(7)^2=49q \quad [11]$$

The *RCP* rR^2 of the ten-man Red infantry section is:

$$rR(0)^2=q(10)^2=100q \quad [12]$$

Since $rR^2(t) > bB^2(t)$, then by Eq. [10], the Red section will eventually win due to its superior RCP because it has more numbers. At the end, the Red section will completely destroy all Blue forces (i.e. B(t)=0). We can calculate the number of surviving

Red forces, by substituting the values into Eq. [7]:

$$bB^2(t)-rR^2(t)=bB^2(0)-rR^2(0) \quad [13]$$

$$0-qR^2(t)=49q-100q \quad [14]$$

$$R(t)=\sqrt{51}=7.14\approx 7 \quad [15]$$

This example implies that in a battle between a seven-man section and a ten-man section of the same quality, the seven-man section will be wiped out (survival rate of 0%) while the ten-man section will lose only three soldiers (with seven soldiers surviving - a survival rate of 70%).

Therefore, the side with superior numbers will lose *fewer* soldiers and has a *higher* survival rate. In a battle between two forces of equal quality, the difference in numbers exerts a significant effect on the outcome.

Test Case 2

Consider a case where both Blue and Red forces are seven-man sections, with Blue forces of quality q and Red forces of lower quality 2/3 q. We want to determine which side wins, and with how many surviving troops.

At the start, the *RCP* bB^2 of the seven-man Blue infantry section is:

$$bB(0)^2=q(7)^2=49q \quad [16]$$

The RCP rR^2 of the seven-man Red infantry section is:

$$rR(0)^2=2/3\ q(7)^2=32.7q \quad [17]$$

Since $bB^2(t)=49q$ is greater than $rR^2(t)=32.7q$, then by Eq. [9], we know that the Blue section will win,

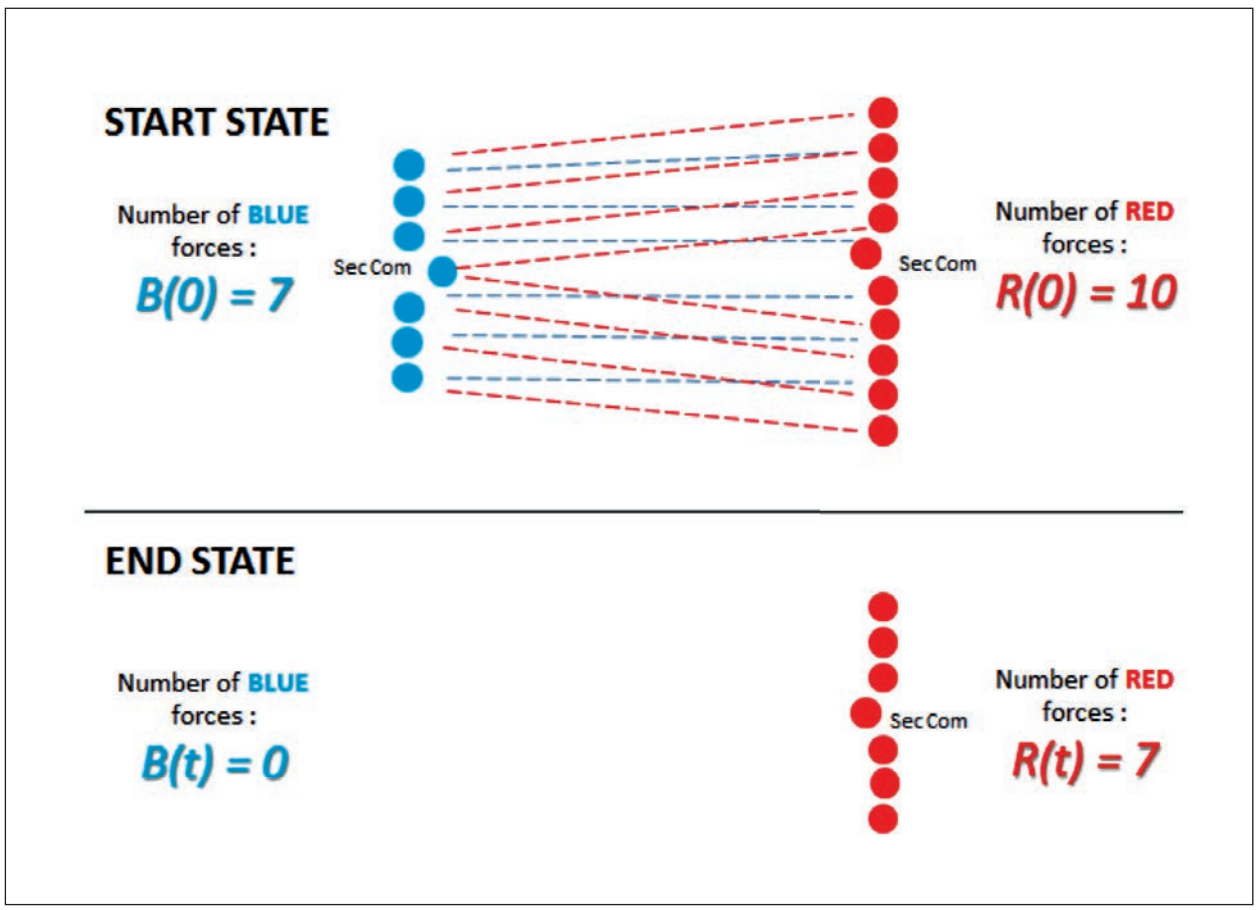

Figure 2: Comparison of Number of Blue to Red Soldiers and Start State and End State.

due to its greater quality. At the end, the Blue section will completely destroy all Red forces. We can calculate the number of surviving Blue forces:

$$bB^2(t)-rR^2(t)=bB^2(0)-rR^2(0) \quad [18]$$

$$qB^2(t)-0=49q-32.7q \quad [19]$$

$$B(t)=\sqrt{16.3}=4.04\approx 4 \quad [20]$$

This implies that in a battle between a seven-man Blue section of quality q and a seven-man Red section of lower quality 2/3 q, the seven-man Red section will be wiped out while the seven-man section will lose three soldiers (with four soldiers surviving—a survival rate of 58%).

As compared to the difference in numbers, the difference in quality between two equally-sized forces has a lesser effect on the outcome of battle. In this example, the survival rate of the side with superior quality is 58%, compared to 70% in the earlier example.

TACTICS FOR BATTLES

These following cases suggest some tactics for commanders:

1. If Blue forces are outnumbered but are of greater quality, they can defeat a Red enemy force of lower quality. Commanders should use Tactics of Concentration—to divide the Red forces into smaller groups, so that Blue forces have greater quantity and quality (and so greater RCP) in each battle against the smaller groups of Red forces. The Blue forces can take on these smaller groups of Red forces one by one, eventually wiping them out.

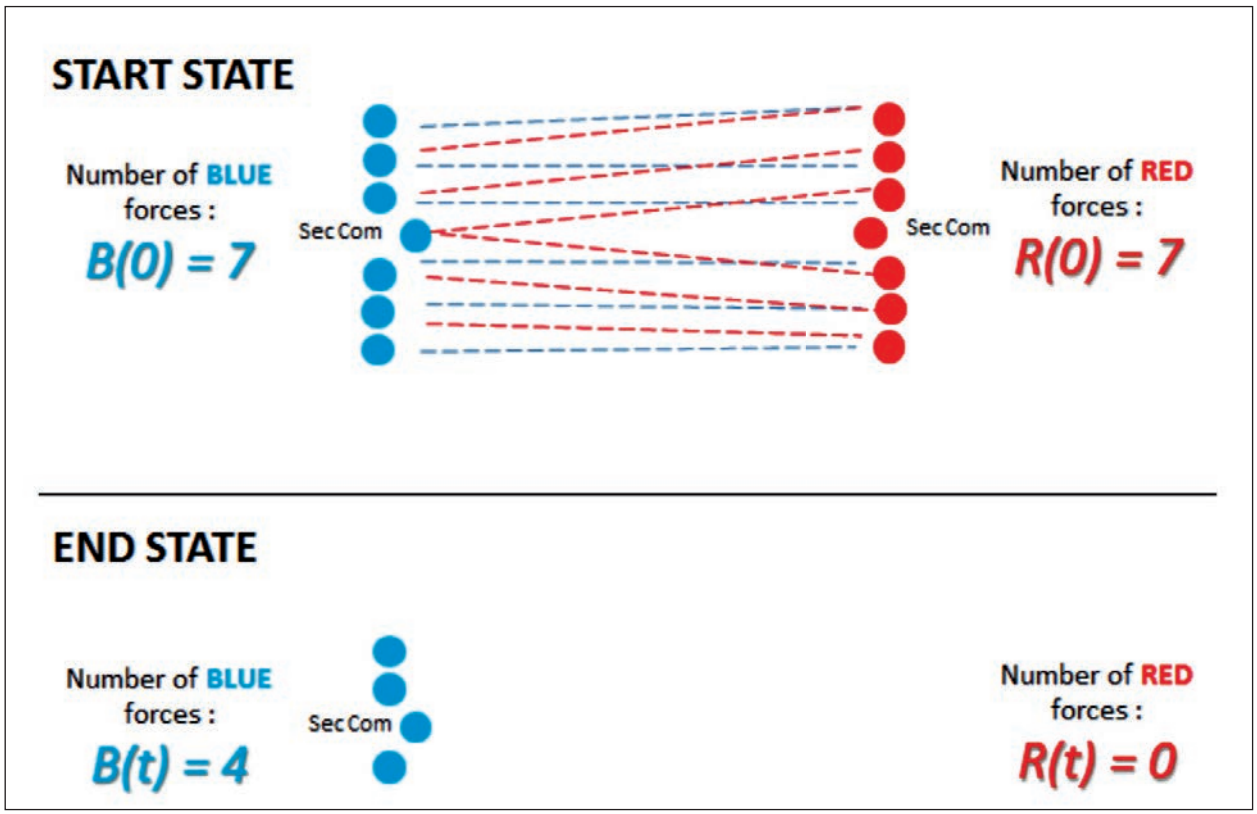

Figure 3: Comparison of Number of Blue to Red Soldiers and Start State and End State.

2. If Blue forces have greater numbers but of lower quality, they can defeat a smaller enemy force of greater quality. Commanders should use Tactics of Numbers—to prevent the Red forces from dividing Blue forces (i.e. prevent Red forces from using Tactics of Concentration against them).

3. If Blue and Red forces are evenly matched in numbers and quality, commanders should not fight the enemy head-on. Instead, commanders should use Tactics of Division—to use terrain, time and location to set up a battle favourable to them. This could involve the deliberate setting of decoys, traps and surprise manoeuvres.

For convenience, these tactics can be summarised in *Table 1* below.

Case-Study: Battle of Trafalgar (1805)

The results of the Lanchester Square Law can be applied to the Battle of Trafalgar (1805), between the British fleet led by Admiral Lord Nelson and the Franco-Spanish fleet led by Admiral Pierre-Charles Villeneuve. At the start, the British fleet had twenty-seven ships and a total of 2,148 naval guns. The Franco-Spanish fleet was greater in numbers, with thirty-three ships and 2,568 naval guns.

We can calculate the RCP of the British to Franco-Spanish fleet at the start of the battle. For simplicity, we assume that the quality of British and Franco-Spanish forces depends on: (1) the number of naval guns; and (2) the quality of crew. Assume further that the Franco-Spanish crew were two-thirds as effective as the British crew. This is reflected in *Table 2* on the next page.

On paper, the Franco-Spanish fleet had superior RCP and would secure a decisive victory. The Franco-Spanish advantage in numbers would compensate for the lower quality. However, history tells us that the outcome of the battle was a decisive British victory with no ships lost (a survival rate of 100%). The defeated Franco-Spanish fleet lost 22 ships (a survival rate of 18.5%).

How did Nelson achieve victory? In the nineteenth century, a common tactic of naval combat involved a naval fleet forming up in a single line, approaching the enemy fleet, also in a single line, then using all the guns on one side to engage the enemy. Both fleets would be in parallel lines. Instead, Nelson bucked the trend. According to Nelson's plan, the British fleet would cut the Franco-Spanish line into three parts. The British ships would approach in two columns of 13 ships and 14 ships, in a direction perpendicular,

Numbers of Blue forces		Quality of Blue Forces		
		Lower quality than Red	**Same quality as Red**	**Greater quality than Red**
	Smaller numbers than Red	Avoid battle	Avoid battle, apply Tactics of Division if unable to avoid battle	Apply Tactics of Concentration
	Same numbers as Red	Avoid battle, apply Tactics of Division if unable to avoid battle	Apply Tactics of Division	Apply Tactics of Concentration
	Greater numbers than Red	Apply Tactics of Numbers	Apply Tactics of Numbers	Engage immediately

Table 1: Classification of Tactics in Different Contexts of Combat.[6]

	British	Franco-Spanish
Quantity (of ships) B(0) and R(0) (in units)	27	33
Quality (of guns and crew) b and r (in units of thousands)	2.148×3=6.4	2.568×2=5.136
RCP bB^2 (0) and rR^2 (0) (in units)	**4,698**	**5,593**

Table 2: Comparison of RCP between British and Franco-Spanish Fleets.

instead of parallel to the Franco-Spanish fleet.[8] This would cut the numerically superior Franco-Spanish line into three parts: a front group, middle group, with the command element, and rear group.

In his commander's guidance, Nelson gave two pointers: (1) to cut-off the enemy's rear; and (2) to concentrate superior force on the rear elements.[9] In the first phase, the two columns of British ships

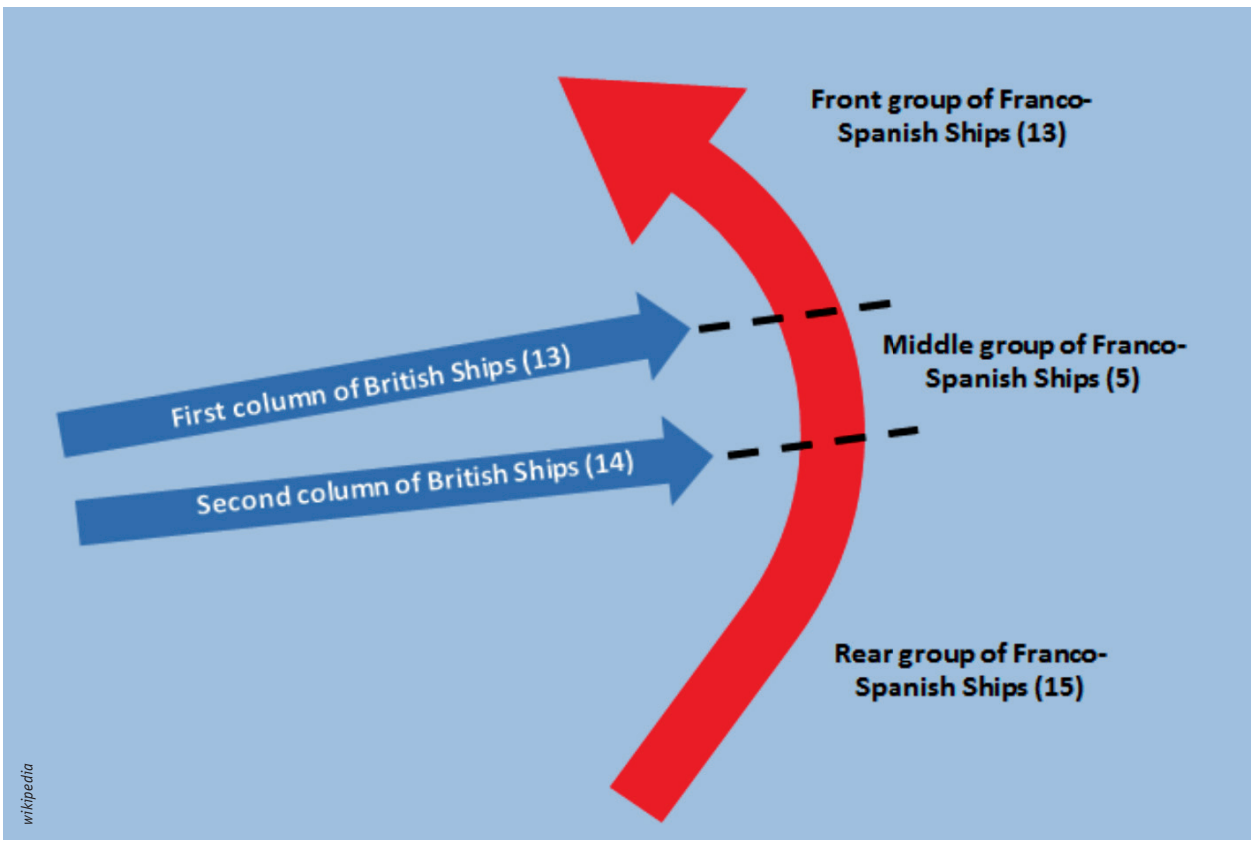

Figure 4: Distribution of British and Franco-Spanish Ships at the start of the Battle of Trafalgar.

	British	Franco-Spanish
Quantity (of ships) B_1 (0)+B_2 (0) and R_1 (0)+R_2 (0)+R_3 (0) (in units)	Column 1: 13 Column 2: 14	Group 1: 13 Group 2: 5 Group 3: 15
Quality (of guns and crews) *b* and *r* (in units of thousands)	2.148×3=6.4	2.568×2=5.136
RCP bB_1^2 (0) + B_2^2 (0) and rR_1^2 (0) + rR_2^2 (0)+ rR_3^2 (0) (in units)	$(6.4)(13)^2+(6.4)(14)^2$ = **2336**	$(5.136)(13)^2+(5.136)(5)^2$ $+(5.136)(15)^2$ = **1864**

Table 3: Comparison of RCP in First Phase of Battle between British and Franco-Spanish Fleets.

would cut the Franco-Spanish line, to isolate the front group of the Franco-Spanish line from the middle group. Separated from the rest, the front group would be forced to re-group, turn back and support the rear elements. The middle group, with the command element, would be isolated from the front and rear groups it commanded, and so command and control would break down. This would allow the second phase to occur—the British could concentrate forces on the rear group of the Franco-Spanish fleet. Nelson's plan would also bring about ship-to-ship combat between the British and Franco-Spanish ships, where the seamanship, faster gunnery and higher morale of each British crew could defeat a Franco-Spanish ship.

We can see the effects of Nelson's tactics on the RCPs of the British and the Franco-Spanish fleets. In the first phase, Nelson's two columns of ships cut up the enemy's line to reduce the total RCP of the enemy ships. *Table 3* above shows that the British tactic to divide up the Franco-Spanish fleet would give the British superior RCP.

In the second phase, one of Nelson's columns with 14 ships, could take on the enemy's rear group of 15 ships. Thus, with almost equal numbers of British and Franco-Spanish ships, this would lead to a ship-to-ship fight. Each British ship could then capitalise on its advantage of superior quality over another Franco-Spanish ship. This is illustrated in *Table 4* on the next page.

Nelson's fleet avoided head-on confrontation with the numerically superior Franco-Spanish fleet. In essence, Nelson's plan to cut-up the enemy into smaller groups illustrates *Tactics of Concentration*. To re-cap, the tactics state that a commander should "divide the Red forces into smaller groups, so that Blue forces will have greater quantity and quality in each battle against the smaller groups of Red forces.

	British	Franco-Spanish
Quantity (of ships) B(0) and R(0) (in units)	14	15
Quality (of guns and crews) *b* and *r* (in units)	3	2
Relative Combat Power bB_1^2 (0) and rR^2 (0) (in units)	**588**	**450**

Table 4: Comparison of RCP in Second Phase of Battle between British 14-ship Column and Franco-Spanish Rear Forces.

The Blue forces can take on these smaller groups of Red forces one by one, eventually wiping them out."[10]

In sum, the results of Lanchester's Square Law can be directly applied to modelling combat engagements. In fact, Lanchester asserted that Lord Nelson, "if not actually acquainted with the Square Law, must have some equivalent basis on which to figure out his tactical values."[11]

Nelson's fleet avoided head-on confrontation with the numerically superior Franco-Spanish fleet. In essence, Nelson's plan to cut-up the enemy into smaller groups illustrates Tactics of Concentration.

Comments on Validity of the Lanchester Square Law

Several authors have attempted to verify the validity of the Lanchester's Square Law. They compared actual results from historical battles such as the Ardennes, Kursk and Iwo Jima and idealised battles at the US Army National Training Centre, against the values predicted by the Square Law. The results indicate that the RCP in actual situations is not bB^2, but closer to bB^{Υ}, where Υ ranged from 1 to 1.5.[12] Many factors in actual combat could explain the deviation, including the lack of fire control and discipline, the arrival of reinforcements, the effect of combined arms and support elements and the effect of the 'ace firer/pilot' (a minority of gunners is attributed the majority of the kills).[13] We should not expect that the Lanchester Square Law will be fully accurate. Rather, it is a useful heuristic to allow commanders to think about the RCP of two forces for quick decision-making in battle.

IMPLICATIONS FOR FORCE STRUCTURE AND FORCE PREPARATION IN SINGAPORE'S CONTEXT

At this juncture, the author highlights that we should take stock of the above discussion, to derive implications for force preparation, structure and training.

Importance of Numbers: Implications for the Singapore Army's Force Structure

The results of the Lanchester Square Law show that numbers give rise to a quality of their own.[14] From the first learning point in *Table 5* below, we see that RCP of a force is bB^2. If the enemy has thrice our numbers, our forces will need to be three-squared (nine) times better in quality to achieve RCP parity.

As Singapore's birth rate falls, we need to adjust our force structures to meet operational outcomes. By 2047, the number of 18-year old males available for conscription will shrink to 29,906, a 20.5% reduction from its peak of 37,619 in 2012.[15] This will affect our force structure build-up during National Service and our Operationally-Ready National Service forces.

The Army has already reduced the size of the section to seven men by compensating with increased firepower. The 20% reduction in the cohort size may prompt us to make further cuts in manpower-intensive areas (e.g. infantry sections and platoons). However, we cannot indefinitely shrink our force structures at the tactical level. In *Table 6* on the next page, I consider the effects of shrinking a seven-man section to six men on the RCP, compared to the RCP of an aggressor ten-man section.

The RCP of the seven-man section is half that of the ten-man section. To achieve parity with the ten-man section, the seven-man section would need to double its quality (through superior training or increased weapon effectiveness). The six-man section would have to triple its quality to achieve parity with the aggressor section. Each soldier of the six-man section will need to be physically fitter, more accurate in aim and exhibit greater teamwork, in order to take out the ten-man section.

Moreover, the effect of casualties on the RCP of a section is significant. One and two seriously-wounded casualties will have major impact. For each casualty, at least one additional soldier will have to provide immediate buddy-level aid. This reduces the RCP of the section dramatically, illustrated in *Table 7* on the next page.

The figures are clear. With one casualty and one additional soldier providing buddy aid, the individual quality of soldiers of the seven-man section has to be four times better than the quality of enemy soldiers. With two casualties and one soldier providing buddy aid, the quality of soldiers has to be seven times higher. In the extreme case of the six-man section

S/N	Theme	Learning Point
1	Lanchester Square Law	The RCP of a force is proportional to the square of the numbers of units and proportional to the quality of units.
2		If we know the qualities and numbers of two forces at the start of a battle, we will know the outcome of the battle.
3	Implications of the Lanchester Square Law	The side with superior numbers will lose fewer soldiers and has a higher survival rate, than the side with fewer numbers (which will be completely eliminated at the end of the battle).
4		The difference in numbers between two forces of equal quality has a larger effect on the outcome of battle, than the difference in quality between two forces of equal numbers.

Table 5: Summary of Learning Points.

	Original Blue Seven-Man Section	Revised Blue Six-Man Section	Red Ten-Man Section
Quantity B(0) and R(0) (in units)	7	6	10
Quality *q* (in units)	*q*	*q*	*q*
RCP $bB^2(0)$ and $rR^2(0)$ (in units)	*49q*	*36q*	*100q*

Table 6: Comparison of RCP between Six-Man, Seven-Man, and Ten-Man Sections.

	Original Blue Seven-Man Section		Revised Blue Six-Man Section		Red Ten-Man Section
Quantity *B*(0) and *R*(0) (in units)	7		6		10
Effective Quantity (with Blue forces incurring	One Casualty (with one buddy)	Two Casualties (with one buddy)	One Casualty (with one buddy)	Two Casualties (with one buddy)	
Casualties) (in units)	5	4	4	3	10
Quality q (in units)	q		q		q
RCP $bB^2(0)$ and $rR^2(0)$ (in units)	25q	16q	16q	9q	100q

Table 7: Comparison of RCP between Six-Man, Seven-Man, and Ten-Man Sections, in the Event of Blue Forces Incurring One or Two Casualties.

with two casualties and one providing buddy aid, the individual quality of soldiers of the six-man section has to be at least ten times higher. This is an unreasonably lofty expectation that even Special Forces soldiers cannot be naturally assumed to attain. Therefore, the results of the Lanchester Square Law show that numbers give rise to a quality of their own. Going forward, the Army will need to exercise caution when changing the force structure of units at the tactical level.

Since the quality of a force directly depends on superior firepower, the SAF must continue to invest in defence capabilities that provide accurate, precise and high volumes of fire.

This leads us to the second point—the reduction in the size of cohort available for enlistment impacts our Operationally-Ready National Service forces. The size of cohort affects both the size of Singapore's resident labour force as well as the manning levels of operationally-ready units, because both draw from the same pool of people. Operationally-Ready units face issues such as competing demands from employers and the increased movement of Singaporeans abroad for study or work during In-Camp Training (ICT) periods. A wise organisation should ensure little change to the personnel of combat groups, so that comrades in peacetime manoeuvres shall be comrades in war.[16] Therefore, NS commanders must ensure a high level of manning and minimal disruption to unit structures by managing the inflow and outflow of NSmen in each ICT cycle.

Importance of Quality: Implications for Force Preparation of Singapore's Operationally-Ready Forces

The second learning point from *Table 5* is: if we know the qualities and numbers of two forces at the start of a battle, we will know the outcome of the battle. So, force preparation is critical, because: (1) how we task-organise and concentrate forces at the start of the battle directly impacts the outcome; (2) and peacetime training establishes a baseline for wartime performance.

The overall quality of forces depends on many factors. In ground combat fire-fights, this may be reduced to superior firepower (weapon effectiveness, accuracy of fire and rate of fire), teamwork and the effective use of terrain. Since the quality of a force directly depends on superior firepower, the SAF must continue to invest in defence capabilities that provide accurate, precise and high volumes of fire. For instance, the vehicle-mounted remote controlled weapon systems (e.g. systems on board the Terrex Infantry Carrier Vehicle) would give our forces superior accuracy of fire and rate of fire, and thus higher quality. Commanders would also have to keep a high level of force preparation before battle and during re-organisation, so that these weapon systems continue to deliver under battle conditions.

The quality of a force also depends on how effectively soldiers are able to use their firepower and their combined teamwork. For NS commanders, we must focus on factors within our control to ensure a high baseline quality during peacetime. This justifies the conduct of the Big Three during In-Camp Training—physical fitness, marksmanship training, combat fitness—in addition to section and platoon-level drills, first-aid and chemical defence training. While quality, compared to quantity may have less contribution to RCP, every commander has a duty to maximise force preparation to strengthen the quality of soldiers. This gives them the best chances of victory in combat.

CONCLUSION

Sun Zi observed that "he will win who knows how to handle both superior and inferior forces."[17] In this paper, the author has shown that the Lanchester Square Law gives us a simple framework to compare the relative combat power (RCP) of two forces that commanders at all levels—including junior commanders—can use. From Eq. [7], the Law shows that the RCP of a force is proportional to the square of the number of the units and proportional to the quality of the units, and the difference between two relative combat powers bB^2 (t) and rR^2 (t) is constant throughout the battle.

The onus is on commanders to maintain a high baseline of quality in peacetime, as Singapore depends heavily on both the quantity and quality of our Operationally-Ready servicemen.

The results of the Lanchester Square Law suggest tactics of Concentration, Numbers and Division that commanders should adopt when dealing with superior and inferior forces. The importance of numbers has implications for the force structure of our army and the advantage of an opponent's numbers needs to be compensated by a significant increase in the quality of our own forces. The onus is on commanders to maintain a high baseline of quality in peacetime, as Singapore depends heavily on both the quantity and quality of our Operationally-Ready servicemen. Only then can we harness the advantages of quantity and quality, enhanced by detailed and coordinated planning, to secure a swift and decisive victory over any aggressor.

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